



Slope (M)

M is the measure of the steepness of the line.

M in mathematics is the rate of change of **y** with respect to **x**.

How do we find Slope?

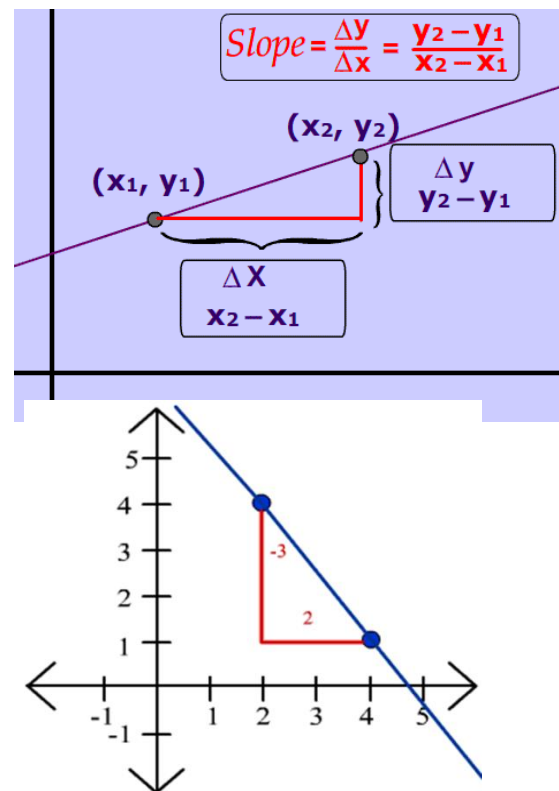
$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

The rise is -3 and the run is 2.

$$\text{Slope} = \frac{-3}{2}$$

The slope for the line above is $-\frac{3}{2}$.

It is a negative slope!



Point-Slope Equations:

To write an equation for a line **L** that is not vertical, it is enough to know its slope **m** and the coordinates of a point **p₁(x₁, y₁)** on it. If **P(x,y)** is any other point on **L**, then **x ≠ x₁**, and we can write the slope **m** of **L** as:

$$m = \frac{y - y_1}{x - x_1} \quad \dots(1)$$

Multiplying both sides of Eq. (1) by **x - x₁**, gives us the more useful equation:

$$y - y_1 = m(x - x_1). \quad \dots (2)$$



Example1: Write an equation for the line that passes through the point (2, 3) with slope $-3/2$.

Solution:

$y - y_1 = m(x - x_1)$ (Start with the general point-slope equation, Eq. (2))

$$y - 3 = -\frac{3}{2}(x - 2) \quad (\text{Take } m = -3/2 \text{ and } (x_1, y_1) = (2, 3).)$$

$$y = -\frac{3}{2}x + 3 + 3 \quad (\text{Solve for } y.)$$

$$y = -\frac{3}{2}x + 6.$$

Example 2: Write an equation for the line through (-2,-1) and (3,4).

Solution: We first calculate the slope and then use Eq. (2).

$$m = \frac{-1 - 4}{-2 - 3} = \frac{-5}{-5} = 1$$

The (x_1, y_1) in Eq. (2) can be either (-2, -1) or (3, 4):

$\frac{\text{With } (x_1, y_1) = (-2, -1)}{y - (-1) = 1 \cdot (x - (-2))}$	$\frac{\text{With } (x_1, y_1) = (3, 4)}{y - 4 = 1 \cdot (x - 3)}$
$y + 1 = x + 2$	$y - 4 = x - 3$
$y = x + 1.$	$y = x + 1.$
Same result	

Either way, $y = x + 1$ is an equation for the line .



SLOPE-INTERCEPT FORM:

Let L is any line intersect y- axis in a point

$(0,b)=(x_1,y_1)$

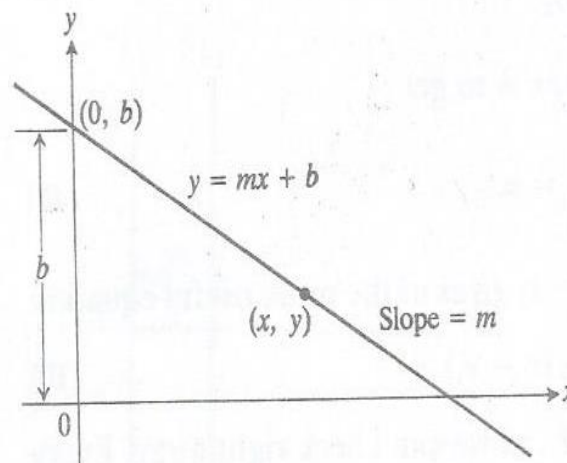
And let (x_2,y_2) another point on L, then

$$M = \frac{y_2 - y_1}{x_2 - x_1}$$

Output **Input**

$$y = mx + b$$

slope **y-intercept**



IF OUR SLOPE = 4 AND THE Y-INTERCEPT IS 3, THEN $y = 4x + 3$.

THE KEY IS HAVING THE "Y" ON ONE SIDE BY ITS SELF.

$$Y = mx + b$$

Example 3:

The standard equation for converting Celsius temperature to Fahrenheit temperature is a slope-intercept equation. The line passes through

the point $(0,32)$ because $F=32$ when $C=0$.

It also passes through the point $(100,212)$

because $F=212$ when $C=100$

The line's slope is:

$$m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$$

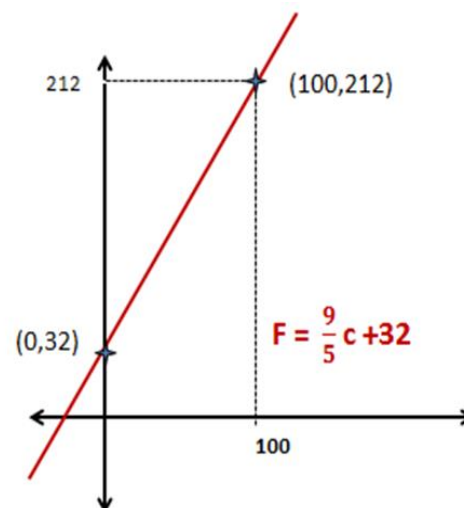
- The F-intercept equation for line is:

$$b = 32$$

- The resulting slope-intercept equation for the line:

$$Y = \frac{9}{5}x + 32$$

$$F = \frac{9}{5}c + 32 \quad \text{and} \quad c = \frac{5}{9}(F - 32)$$





Example 4: Find the slope and y-intercept of the line $8x+5y=20$.

Solution: Solve the equation for y to put the equation in slope-intercept form. Then read the slope and y-intercept from the equation:

$$8x+5y=20$$

$$5y = -8x+20$$

$$y = -\frac{8}{5}x + 4.$$

The slope is $m = -8/5$. The y-intercept is $b = 4$.

Horizontal and Vertical Lines:

The standard equations for the horizontal and vertical lines through a point (a, b) are simply $y=b$ and $x = a$. A point (x, y) lies on the horizontal line through (a, b) if and only if $y = b$. It lies on the vertical line through (a, b) if and only if $x = a$.

Slope

1. The slope of the line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, $x_1 \neq x_2$, is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}.$$

2. $m = \tan \phi$ (ϕ is the angle of inclination).
3. Vertical lines have no slope.
4. Horizontal lines have slope 0.
5. For lines that are neither horizontal nor vertical it is handy to remember:
 - a) they are parallel $\Leftrightarrow m_2 = m_1$;
 - b) they are perpendicular $\Leftrightarrow m_2 = -1/m_1$.(The symbol $\Leftrightarrow$ is read "if and only if.")



Equations for Lines

$x = a$	Vertical line through (a, b)
$y = b$	Horizontal line through (a, b)
$y = mx + b$	Slope–intercept equation
$y - y_1 = m(x - x_1)$	Point–slope equation
$Ax + By = C$	General linear equation (A and B not both zero)

Lines That Are Parallel or Perpendicular:

Parallel lines have equal angles of inclination. Hence, if they are not vertical, parallel lines have the same slope. Conversely, lines with equal slopes have equal angles of inclination and are therefore parallel .

If two **parallel** lines L_1 and L_2 , their slopes m_1 and m_2 are related by the equation **$m_1 = m_2$** .

If neither of two **perpendicular** lines L_1 , and L_2 is vertical, their slopes m_1 and m_2 are related by the equation **$m_2 = -\frac{1}{m_1}$** and **$m_1 m_2 = -1$** .

Example 5:

The slope of a line perpendicular to a line of slope $3/4$ is $-4/3$.